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A fast and accurate method for detecting fingerprint reference point

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Abstract Unique and stable reference point is essential for registration and identification in automated fingerprint identification systems. Most existing methods for detecting reference points need to scan the fingerprint image or orientation field pixel by pixel or block by block to confirm a candidate reference point. The inherent complexity of this process makes those methods time-consuming. In this paper, we proposed a two-step method to improve the efficiency of detecting reference points by (i) determining the singular point, i.e. the approximate position of the reference point, in a novel fast way; then (ii) refining the reference point precisely in the local area of the singular point. In the first step, a *walking algorithm* is proposed which can walk directly to the singular point without scanning the whole fingerprint image and hence it is extremely fast. Then, in the local area around the singular point, an enhanced method based on mean-shift concept (EMS-based method) is designed to localize the reference point precisely. Experimental results on FVC2000 DB1a and DB2a databases validate that the proposed WEMS (Walking+EMS) method outperforms two state-of-the-art methods in terms of accuracy and efficiency.

Keywords Fingerprint · Reference point · Walking · Mean-shift

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1 Introduction

As one of the most reliable biometric personal identification techniques (e.g., face recognition [32, 33], iris recognition [29] and finger-vein recognition [6, 18]), fingerprint recognition [21, 30] has been extensively used in security control or personal authentication. Generally, there are two categories of features for the fingerprint: global features like ridge flow pattern and local features like minutia. Singular points (upper core, lower core and delta point), where the orientation field is discontinuous or the ridge curvature is the highest, are typical global markers. They are employed to align two fingerprints in order to speed up the matching process. Unfortunately, the absence of singular points in arch type fingerprints leads to the failure of aligning an arch type one with the others. Therefore, reference points defined in all types of fingerprints have attracted a lot of attentions and two widely used strategies to detect reference points are (i) employing two methods separately on non-arch and arch type fingerprints [19, 23] and (ii) using the unique characteristic of reference points to design a detection method for all kinds of fingerprints [1, 14, 27]. The former one needs the classification for fingerprints to choose the detection method for the corresponding type and thus it is more complicated. While the latter usually suffers from less accuracy because of leveraging the characteristic of reference points in arch type fingerprints. In the literature, the reference points only defined in non-arch type fingerprints are known as singular points. To avoid ambiguity, in this paper the reference point is specifically referred to that is defined in all types of fingerprints.

This paper aims to explore a strategy of achieving the best tradeoff between efficiency and accuracy. We propose to combine two methods (walking algorithm and EMS-based method) more tightly to achieve this goal. The proposed walking algorithm, aiming at detecting the rough position of the singular point (upper core) in non-arch type fingerprints by walking directly to it on a block-wise Extended Directional Field (EDF) without scanning the whole image, is superior in efficiency. Suppose it consumes constant time t_0 to check if one candidate pixel is a singular point and the scanning process visits N_0 pixels, then the total time of detecting the singular point for each image will be $N_0 \cdot t_0$. Our walking algorithm only needs to visit tens of pixels (reducing N_0 approximately by 10 times compared to traditional scanning-based methods), and the time t_0 is only comprised of some comparison operations, resulting the average processing time dropping down to less than 0.1 ms. The other method, the enhanced mean-shift-based (EMS-based) method, is proposed by modifying the mean-shift-based method [1] to a version with much higher accuracy which can detect reference points in all types of fingerprints. Our strategy is that after determining the rough position of singular point we employ EMS-based method on the local area of the singular point to refine the position and if no singular point is detected by walking algorithm, the EMS-based method will be employed on the whole fingerprint image. Theoretically the proposed WEMS (Walking+EMS) method will benefit on both the efficiency of walking algorithm and the accuracy of EMS-based method.

The contributions of this paper are summarized as follows.

- A new efficient singular point detection method called walking algorithm is proposed. It is very fast because it can walk directly to the singular point by

following the designed heuristic direction, which is quite different from traditional scanning-based methods.
- An accurate reference point detection method called EMS-based method is proposed. This method as an enhanced version of the existing mean-shift-based method [1] can work well in non real-time fingerprint recognition systems.
- By combing these two methods, a fast and accurate fingerprint reference point detection method is introduced in this paper. The proposed method benefits from the efficiency of the walking algorithm and the accuracy of the EMS-based method and can be used to improve the performance of real-time fingerprint recognition systems.

The remaining of this paper is organized as follows. Section 2 introduces the related work. Then the walking algorithm for detecting singular point in non-arch type fingerprints is introduced in Section 3. Section 4 describes the EMS-based method and the combined WEMS method for reference point localization. In Section 5, some experimental results are presented. We finish with conclusions and future works in Section 6.

2 Related work

Singular point detection methods can be categorized into Poincaré index (PI) based techniques [3, 16, 28, 31], model-based techniques [8, 24], methods based on local characteristics of orientation field [3, 5, 15, 26], and others [11, 12]. The PI of a point is defined as the accumulative orientation differences along a closed path surrounding the point. The values of PI for core, delta and non-singular point are π, $-\pi$ and 0 respectively. PI-based methods calculate PI for each point to determine the singular points according to the value of PI. That is, they need to scan the fingerprint image pixel by pixel (or sampling with interval of several pixels). The model-based method in [8] used zero-pole model for orientation field to derive a spacial relationship between singular points and their neighbor points, then a Hough Transform method was utilized to detect singular points. Hough transform also needs to scan all pixels to fill the parameter space. Qi and Liu [24] proposed a zero-pole model to define a new Angle Matching Index (AMI) and then the AMI of each pixel was calculated to detect singular points. Methods based on local characteristics of orientation field explore the orientation image regions characterized by high irregularity [5, 26], curvature [15], or symmetry and the explored regions should cover the whole image. Other methods [5, 11, 12] could not avoid the scanning process either. In this paper we deal with this issue by designing a heuristic direction for each pixel on the orientation field and then following the direction to walk directly to the singular point.

However, the singular points are not present in arch type fingerprints. So reference points for all types of fingerprints have been extensively researched [1, 7, 14, 17, 27]. Liu et al. [17] developed an effective algorithm to locate an unique reference point for all types of fingerprints based on multi-scale analysis of the orientation consistency to search for the local minimum. The method of Le and Van [14] is similar to Liu's, taking the point with maximum Variation and Symmetry Combined Energy (VSCOME) as the reference point. Both of these methods have to filter delta point or lower core point, which inevitably increases the complexity. Tams [27] used a pre-trained tented arch model to fit the orientation field of fingerprint and defined the position of model's core as the reference point. But for arch type fingerprint, this method can not work effectively. Areekul and Boonchaiseree [1] defined the focal point as the unique reference point and introduced the mean-shift-based algorithm to localize it. Although these reference points have different definitions and slightly distinct positions, they will locate at the same local area of singular point (upper core) for non-arch fingerprints and at the region with maximum curvature for arch type fingerprints. This makes it feasible to localize the reference point in the local area of singular point after the singular point has been found by the fast walking algorithm. While the reference point detection method should be accurate and can work in local area, so we choose the mean-shift-based method [1]. Furthermore, we improve the accuracy of this method by (i) using orientation field with a block size of 8×8 instead of 16×16 and (ii) assigning a weight to each cross point and use the weighted average point of cross points as the reference point. The version of mean-shift-based method with accuracy improved is termed EMS-based method.

In short, our work in this paper combines the efficiency of a singular point detection method (walking algorithm) and the accuracy of a reference point detection method (EMS-based method). The details of the proposed method will be introduced as follows.

3 Singular point determination

3.1 Analysis of orientation field

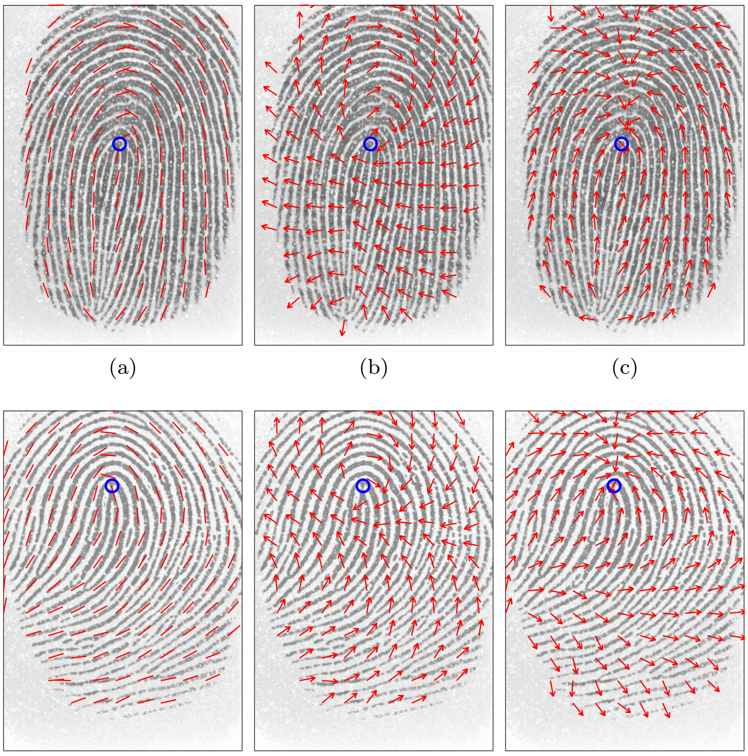
The orientation field is extensively used to detect reference points, and there are many methods for orientation field estimation, such as gradient-based methods [10, 22, 35, 36], model-based methods [4, 13, 25] and others [9, 37]. In this paper, the orientation field Θ of fingerprint image is estimated in block-wise with block size $b \times b$ ($b = 8$ in our study) using method in [37] which is robust to noise.

Let I represent the fingerprint image with size $w_0\times h_0$ in grey level, where $I(u, v)$ denotes the pixel value at uth row and vth column with $1 \le u \le h_0, 1 \le v \le w_0$. Suppose the size of Θ is $h \times w$, then we have $h = \lfloor h_0/b\rfloor$ and $w = \lfloor w_0/b\rfloor$ where $\lfloor\cdot\rfloor$ is the floor operator. Let $\Theta(i, j)$ $(1 \le i \le h, 1 \le j \le w)$ denote the orientation of the block centered at $(i\cdot b+b/2, j\cdot b+b/2)$ of I. Then the Squared Directional Field [2], SDF, can be obtained by doubling $\Theta(i, j)$ at each block, i.e. $SDF(i, j) = 2\cdot\Theta(i, j)$, as illustrated in Fig. 1b and Fig. 1e.

It is easy to note that the singular point is surrounded by some clockwise approximate circles in the local area of SDF. Therefore, if each direction in SDF minus $\pi/2$, as shown in Fig. 1c and Fig. 1f, the singular point will be pointed to by all surrounding directions of the new directional field. We call this new directional field as Extended Directional Field (EDF). So the heuristic direction $EDF(i, j) \in [0, 2\pi)$ can be calculated by

$$EDF(i,j) = \left(2\Theta(i,j) - \frac{\pi}{2}\right) \mod 2\pi \tag{1}$$

Furthermore, from any point on EDF we can always walk to either the singular point or the outside of the fingerprint foreground, which inspires us to design a



(a) (b) (c)

(d) (e) (f)

Fig. 1: Examples of orientation fields, SDFs and EDFs. The singular points are marked with blue circles. (a)-(c) Orientation field, SDF and EDF derived from the same fingerprint. (d)-(f) Orientation field, SDF and EDF derived from another fingerprint

significantly fast algorithm for detecting the singular point by walking from some points to it, as shown in next subsection.

3.2 Walking algorithm

In Section 3.1, we introduced a novel directional field EDF and noticed that from certain point we could walk directly to the singular point. Here are the details of walking scheme.

Suppose the origin of the image coordinate system is the left top corner and a point with coordinate (x, y) corresponds to the position of yth row and xth column for matrix EDF. Suppose the start point is $P_0 = (x_0, y_0)$ and the direction at P_0 on EDF is

$$\alpha_0 = EDF(y_0, x_0) \tag{2}$$

then the new position $P_1 = (x_1, y_1)$ after walking one step can be calculated by

$$(x_1, y_1) = \begin{cases} (x_0, y_0 - 1) & \text{if } \frac{\pi}{4} \leq \alpha_0 < \frac{3\pi}{4} \\ (x_0 - 1, y_0) & \text{if } \frac{3\pi}{4} \leq \alpha_0 < \frac{5\pi}{4} \\ (x_0, y_0 + 1) & \text{if } \frac{5\pi}{4} \leq \alpha_0 < \frac{7\pi}{4} \\ (x_0 + 1, y_0) & \text{otherwise} \end{cases} \tag{3}$$

Keep the walking process to the kth point $P_k = (x_k, y_k)$, and collect all these points to form a road, i.e. $Road = \{P_0, P_1, \cdots, P_k\}$. Then if P_k has the same coordinate with P_{i_0}, $i_0 < k$, the walking process comes to the end and we call *Road* is *closed*. The center point P_c of the road from P_{i_0} to P_k serves as a candidate singular point, calculated by

$$\begin{aligned} x_c &= \frac{1}{k - i_0 + 1} \sum_{j=i_0}^{k} x_j \\ y_c &= \frac{1}{k - i_0 + 1} \sum_{j=i_0}^{k} y_j \end{aligned} \tag{4}$$

where (x_c, y_c) gives the position of P_c, (x_j, y_j) is the coordinate of P_j, which is on the walking road *Road*, and (x_{i_0}, y_{i_0}) is the coordinate of point P_{i_0} which equals to (x_k, y_k).

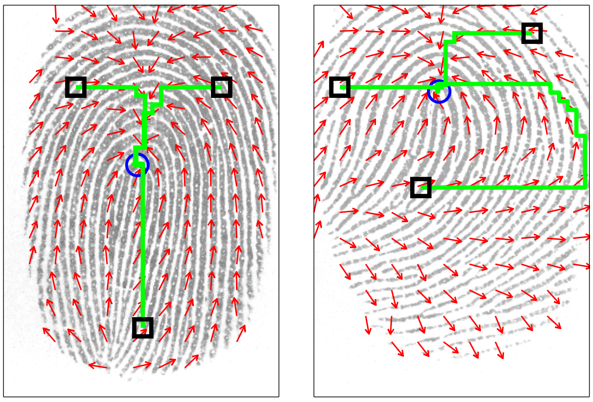
However, it is possible to walk to the background of fingerprint images, in which case we declare that the walking process started with the given point P_0 comes into a failure. The walking process from a given start point is summarized in Algorithm 1.

Algorithm 1: Walk once from a given start point.

Input: Extended directional field EDF, start point P_0
Output: Detected singular point P_c
1 Initiate $Road = \{P_0\}$ and $k = 1$;
2 Get next point P_k according to Eq. (3);
3 Add P_k into *Road* and set $k = k + 1$;
4 Repeat Step 2-3 until *Road* is *closed* or P_k is not in the foreground;
5 Calculate P_c using Eq. (4) and output the result;

Examples of the walking process started with some given points on EDF are depicted in Fig. 2. Apparently, if we can walk to the singular point from a proper start point, the walking road will not be too long. While each step just need very simple calculation as described above, the whole process should be significantly fast.

To improve the probability of success to detect the singular point, it is a natural thought of running Algorithm 1 repeatedly with different start points. Here we give a simple strategy of start points selection by sampling points along x-axis with the step length $d_x = \lfloor w/(n+1) \rfloor$, and along y-axis with the step length $d_y = \lfloor h/(n+1) \rfloor$, where n denotes the number of points to be sampled along x-axis or y-axis ($n = 3$ in our study) and $h \times w$ is the size of block-wise EDF. So we



(a) (b)

Fig. 2: Examples of walking from the given start points on EDFs. Ground truth singular points are marked with "◯", start points are marked with "□"

can build a set S as

$$S = \{(x, y) \big| x = 1 + i \cdot d_x,\ y = 1 + j \cdot d_y,\\ M(y, x) = 1,\ i, j = 1, 2, \cdots, n\} \tag{5}$$

where $M(y, x) = 1$ denotes that (x, y) is in the foreground (M is estimated by method in [35] and has the same size with EDF). Therefore the number N of points in S is not more than n^2, i.e. $N \le n^2$. In fact, if start points are chosen randomly, the performance of the walking algorithm will not have much difference on average but will vary in different trials. More possible strategies for choosing start points will be part of our future works.

At last, for each point in S, we run the Algorithm 1 and get a result, if the result is in the foreground we use the result as the final singular point. If no resulting point is in the foreground after all start points in S are tried, the walking algorithm ends in failure. The walking algorithm is summarized in Algorithm 2.

Algorithm 2: Walking algorithm.

Input: Extended directional field EDF
Output: Singular point P_c
1 Build S according to Eq. (5);
2 Choose a start point P from S;
3 Walk once to get P_c using P and EDF (see Algorithm 1);
4 Repeat Step 2-3 until P_c is in the foreground or no start point left.

The proposed walking algorithm is very simple and can be conveniently implemented. And most importantly, this algorithm is extremely efficient, which allows us to use other accurate but time-consuming method to localize the reference point in the local area around the determined singular point to get a new method with high efficiency and accuracy. On the other hand, the walking algorithm is not capable of dealing with arch type fingerprints, so the chosen method to be combined with the walking algorithm should be able to detect reference point for arch type fingerprints. The next section introduces our EMS-based method and the combined WEMS method for localizing the reference point.

4 Reference point localization

4.1 EMS-based method

The focal point is one of the stable reference points and a classic method for focal point detection is the termed mean-shift-based algorithm in [1] which employed the mean-shift concept with controlling effective area using block-wise orientation field with block size 16×16. To improve the accuracy, we improved mean-shift-based algorithm from the following two aspects: (i) use orientation field with a block size of 8×8 instead of 16×16; (ii) assign a weight to each cross point and use the weighted average point of cross points as the reference point. The motivation of the second improvement is that the cross point generated from convex ridge (whose shape like $\cap$) has more contribution to the final reference point and the relative distance between the cross point and the blocks who generate it has negative correlation to the contribution. So we define the weight of cross point Q as

$$w(Q) = \begin{cases} 1 & \text{if } Q < P_1, P_2 \\ 1 + K \cdot e^{-\lambda R(Q)} & \text{otherwise} \end{cases} \tag{6}$$

where P_1 and P_2 are the center points of blocks who generate the cross point Q. And $Q < P_1, P_2$ means that the y-coordinate of Q is smaller than that of P_1 and P_2, representing a concave ridge. $K, \lambda \in \mathbb{R}$ ($K = 20, \lambda = 0.06$ in the experiment), and $R(Q)$ represents the radius of curvature, approximately calculated by

$$R(Q) = \max\left(\left\|\overline{QP_1}\right\|, \left\|\overline{QP_2}\right\|\right) \tag{7}$$

Fig. 3 illustrates the progress of generating a cross point.

The cross point with a weight is termed weighted cross point and the modified mean-shift based algorithm with accuracy improved is termed EMS-based (Enhanced Mean-Shift-based) method. The main steps of EMS-based method is summarized as follows. Note that all coordinates of points in this method are in pixels.

Step 1 Calculate all weighted cross points from the whole orientation field and find the block Ω with maximum density. Then the initial candidate reference point is obtained by

$$\begin{aligned} x_R &= \frac{\sum_{Q \in \Omega} x_Q \cdot w(Q)}{\sum_{Q \in \Omega} w(Q)} \\ y_R &= \frac{\sum_{Q \in \Omega} y_Q \cdot w(Q)}{\sum_{Q \in \Omega} w(Q)} \end{aligned} \tag{8}$$

where (x_R, y_R) is the coordinate of the candidate reference point, Q is a cross point, (x_Q, y_Q) is the coordinate of Q and $w(Q)$ is defined as Eq. (6).

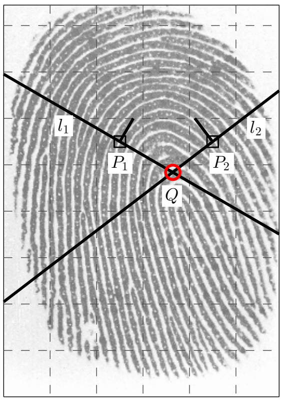


Fig. 3: An example of generating a cross point. P_1, P_2 are center points of two different blocks, l_1, l_2 are straight lines that are through P_1, P_2 and perpendicular to the orientations of two blocks respectively, and l_1, l_2 cross at Q, marked with red circle

Step 2 Determine the effective area to be a top-half circle area, whose center is the block holding the candidate reference point and radius is R (in blocks, $R = 10$ in our study).

Step 3 Use the blocks in the effective area to calculate weighed cross points, then the new reference point is calculated using Eq. (8) by replacing Ω with fingerprint foreground, i.e. taking the weighted average point of all weighted cross points as the new reference point.

Step 4 Calculate the shifted distance, $\delta(t+1)$, between the current reference point $P_r(t)$ and the new reference point $P_r(t+1)$, and the cumulative shifted distance, $S_\delta(t+1)$, by

$$\begin{aligned} \delta(t+1) &= \|P_r(t+1) - P_r(t)\| \\ S_\delta(t+1) &= S_\delta(t) + \delta(t+1) \end{aligned} \tag{9}$$

Step 5 Give the threshold δ_T, S_T ($\delta_T = 3$ and $S_T = 100$ in our study), then if $S_\delta(t+1) > S_T$, the algorithm comes into a failure, and if $\delta(t+1) < \delta_T$, use $P_r(t+1)$ as the final reference point; otherwise, take $P_r(t+1)$ as the candidate reference point and go to **Step 2**.

4.2 WEMS method

In our EMS-based method, we utilize 8×8 blocks to improve the accuracy, which in turn will cost much more processing time and most time is spent on generating weighted cross points from the whole image in **Step 1**. Thus the walk algorithm can be utilized to initialize the candidate reference point, so as to speed up the EMS-based algorithm.

Therefore, a fast and accurate method for reference point detection can be constructed prospectively by combining the proposed walking algorithm and the

EMS-based method. The combined method, termed WEMS method, is summarized as follows. Use Algorithm 2 to locate the singular point (singular block), then use the center of the block as the initial candidate reference point and employ **Step 2-5** of EMS-based method to determine the final reference point. If no singular point can be found by Algorithm 2 (implying the arch type fingerprints), the complete EMS-based algorithm is utilized to localize the final reference point.

According to the above combining strategy, we can predict the WEMS method is as accurate as but faster than EMS-based method thanks to the walking algorithm part. Moreover, other accurate methods that can independently work at local area can be combined with our walking algorithm to improve the efficiency.

5 Experimental results

The proposed method was evaluated on FVC2000 DB1a and DB2a [20], each of which consists of 100 fingers with 8 imprints per finger, total 800 fingerprints. The DB1a and DB2a have the following features:

1) The fingerprints are mainly from 20 to 30 year-old students (about 50% male).
2) Up to four fingers were collected for each volunteer.
3) The images were taken from untrained people in two different sessions and no efforts were made to assure a minimum acquisition quality.
4) All the images from the same individual were acquired by interleaving the acquisition of the different fingers.
5) The presence of the fingerprint cores and deltas is not guaranteed since no attention was paid on checking the correct finger centering.
6) The sensor platens were not systematically cleaned.
7) The acquired fingerprints were manually analyzed to assure that the maximum rotation is approximately in the range $[-15°, 15°]$ and that each pair of impressions of the same finger have a non-null overlapping area.

The proposed method is compared against the mean-shift-based method [1] and the model-based method [27]. The former method uses 16×16 block orientation field to locate the focal point as the reference point and the latter employs a tented arch model to estimate a directed reference point. Our evaluation was carried out on a PC with Intel(R) Core(TM) Processor (i5-3470, 3.2 GHz), 4 GB of RAM, and the methods were coded in C++.

The reference points (besides singular points) usually cannot be marked by hand to determine ground truth. Since the main purpose of the reference points is to align two different fingerprints, then if two fingerprints have been perfectly aligned by hand, the distance of two reference points in these two fingerprints should be zero. Therefore we measure the distance error (DE) as follows. For each Finger, we choose one of the 8 imprints as the template, then compute affine parameters (translation and rotation parameters) of the other 7 ones when they are aligned to the template separately. So the detected reference point in 7 non-template imprints can be projected to the template. We take the average point of total 8 reference points in the template (7 of them are projected from the other imprints) as the true reference point. The DE is finally defined as the Euclidean distance between each reference point and the true reference point.

Like Liu's experiments [17], the performance of different methods for reference point detection is evaluated by the accuracy and consistency. If DE is no more than 10 pixels, the localization for the corresponding reference point is considered to be accurate. If DE is between 10 pixels and 20 pixels, it is considered as a small error. If DE is between 20 pixels and 40 pixels, it is considered as a significant error. If DE is larger than 40 pixels, the reference point is considered to be spurious and should be discarded.

The consistency is measured by the standard deviations of the detected reference points. Let P_{ij} $(1 \leq i \leq 800, 1 \leq j \leq 8)$ denote the detected reference point of the jth imprint from the ith finger and P'_{ij} be the reference point on the template of the ith finger projected from P_{ij}. Then $\frac{1}{8}\sum_{j=1}^{8} P'_{ij}$ is defined as the true reference point for the ith finger as described before. The standard deviation of 8 reference points of the ith finger is given by

$$\sigma_i = \sqrt{\frac{1}{8}\sum_{j=1}^{8}\left\|P'_{ij} - \frac{1}{8}\sum_{j=1}^{8} P'_{ij}\right\|^2} \tag{10}$$

and the average standard deviation of the whole test database is

$$\sigma = \frac{1}{800}\sum_{i=1}^{800}\sigma_i \tag{11}$$

5.1 Comparison of our methods

If the center point of the detected block by the walking algorithm is considered as the final reference point, we can compare the performance of the walking algorithm, EMS-based method and the combined WEMS method to display the effectiveness of the combination. The accuracy, consistency and average processing time of our methods are tested on FVC2000 DB2a and listed in Table 1. The accuracy is defined as the ratio of the number of fingerprints whose DE is not larger than 20 pixels to the number of all fingerprints in the database, i.e. 800. The consistency is measured by average standard deviation (see Eq. (11)), and the average time does not include preprocessing time like segmentation and orientation field estimation. As we can see, our walking algorithm is weak on accuracy but the average time is about 0.04 ms, which is negligible even for real time applications. This extreme efficiency of the walking algorithm allows us to choose more complex but accurate methods to refine the position of the reference points. As an example, by combining the walking algorithm and the EMS-based method, both accuracy and efficiency of the combined WEMS method are improved. As expected WEMS method saves near 3 times processing time than EMS-based method with the accuracy remained.

When the allowed distance error changes from 0 pixels to 40 pixels, the accuracy rate curve can be drawn and illustrated in Fig. 4 where the accuracy rate at allowed distance error e is defined as the ratio of the number of fingerprints whose DE is not larger than e pixels to the number of all fingerprints in the database, i.e. 800 (refer to [14] for more details). As expected the EMS-based method and the WEMS method nearly have the same accuracy for different allowed distance errors,

Table 1: Accuracy, average standard deviation (ASD) and average precessing time (AT) of our algorithms on FVC2000 DB2a

Methods	Acc. (%)	ASD	AT (ms)
Walking	91.13	7.26	**0.04**
EMS-based	98.50	5.14	13.62
WEMS	**98.75**	**4.60**	4.88

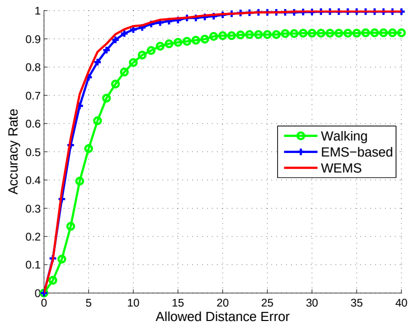


Fig. 4: Accuracy rate curves of our algorithms on FVC2000 DB2a

but the latter has advantages on efficiency (see Table 1). However, the walking algorithm seems not outstanding for any allowed distance error. One reason is that our walking algorithm is not designed to be accurate but to be efficient. Another reason is that the walking algorithm can only detect the singular point for non-arch type fingerprints. Thus it's unfair to compare it with the other two methods which can detect reference points defined in all types of fingerprints.

5.2 Comparison with existing methods

Our combined WEMS method is compared with existing reference point detection methods: mean-shift-based method [1] and model-based method [27]. The accuracies, consistencies and average processing time of different methods on FVC2000 DB1a and DB2a are shown in Table 2 and Table 3 respectively. Our WEMS method outperforms the others in terms of accuracy and consistency and is comparable to mean-shift-based method in terms of average processing time (efficiency). Since the average processing time of our method is only slower than mean-shift-based method by less than 0.5 ms, we can still draw the conclusion that the proposed WEMS method has the best performance.

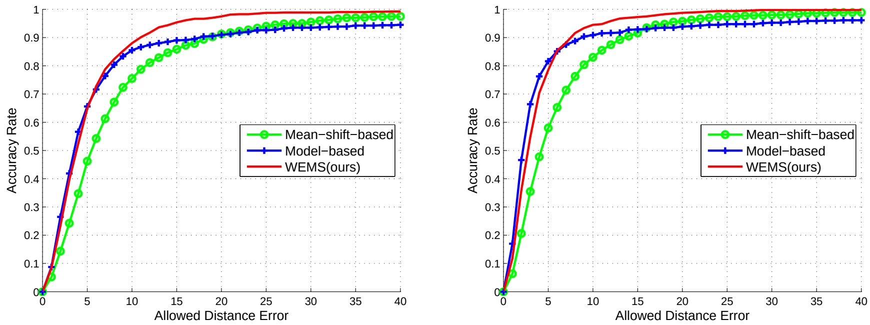
The accuracy rate curves of these three methods are illustrated in Fig. 5. When the allowed distance error is less than 7 pixels, our combined method has similar accuracy rate with model-based method and has higher accuracy rate than means-shift-based method. And when the allowed distance error is more than 7 pixels, the proposed method achieves sustainedly higher accuracy than the others. Since most fingerprint matching methods can tolerate a distance error of 10 pixels,

Table 2: Accuracy, average standard deviation (ASD) and average precessing time (AT) of methods on FVC2000 DB1a

Methods	Acc. (%)	ASD	AT (ms)
WEMS(ours)	**97.50**	**6.32**	5.14
Mean-shift-based [1]	91.25	10.73	**4.78**
Model-based [27]	91.00	8.35	724.39

Table 3: Accuracy, average standard deviation (ASD) and average precessing time (AT) of methods on FVC2000 DB2a

Methods	Acc. (%)	ASD	AT (ms)
WEMS(ours)	**98.75**	**4.60**	4.88
Mean-shift-based [1]	95.75	7.82	**4.69**
Model-based [27]	93.88	7.28	694.47



(a) (b)

Fig. 5: Accuracy rate curves of mean-shift-based method [1], model-based method [27] and our combined method on (a) FVC2000 DB1a and (b) FVC2000 DB2a

the proposed method is possible to improve the matching accuracy of fingerprint identification system, which will be demonstrated in Section 5.3.

5.3 Evaluation through identification performance

Until now the outperformance of the proposed method (Walk+EMS) has been demonstrated through Section 5.1 and 5.2. Considering the role of reference points in fingerprint identification system, a state-of-the-art minutiae-based matching method [34] is applied to indirectly evaluate the proposed WEMS method. Concretely, the matching method aligns two sets of minutiae patterns relative to the coordinates of the reference points detected by different methods. Then the highest similarity score is computed when one set of minutiae is rotated by varied angles relative to its reference point. Eventually the identification performance like ROC (Receiving Operating Curve) is obtained after matching each pair of fingerprint images in the dataset. Theoretically the reference point detected by a more accurate method should correspond to the higher identification performance.

We follow the protocol used in FVC2000 [20] to evaluate the identification performance. Each pair of imprints from the same finger corresponds to a genuine matching score and total 2800 scores for each dataset are collected to form vector **gms**. Then the first imprints from different fingers are matched against each other and the corresponding imposter matching scores are stored in vector **ims**. If any participant in the matching process don't owns a valid reference point, the resulting matching score will be set to 0. Accordingly, FMR (False Match Rate) and FNMR (False Non-Match Rate) are defined by

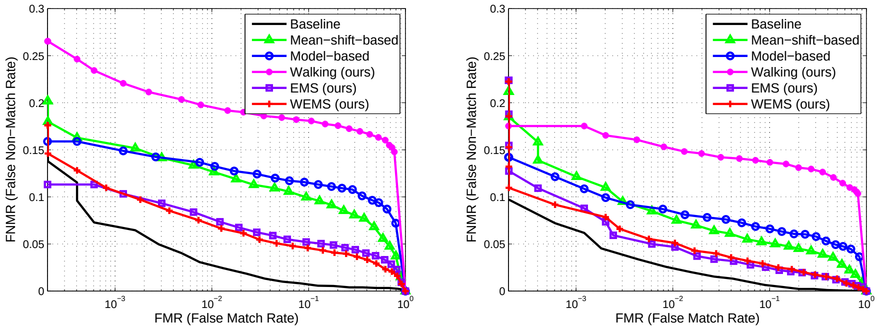
$$\mathrm{FMR}(t) = \frac{\text{Number of sores in } \mathbf{ims} \geq t}{\text{Number of imposter matching}} \tag{12}$$

$$\mathrm{FNMR}(t) = \frac{\text{Number of sores in } \mathbf{gms} < t}{\text{Number of genuine matching}} \tag{13}$$

where the number of imposter matching is 4950 and number of genuine matching is 2800 for FVC2000 DB1a and DB2a. The ROC is given where FNMR is plotted as a function of FMR.

Fig. 6 illustrates ROCs corresponding to mean-shift-based method [1], model-based method [27] and the proposed methods on FVC2000 DB1a and DB2a. The baseline is the ROC of the matching method without pre-alignment. Firstly, the performance of the same method tested on DB2a is consistently better than that on DB1a, because the image quality of DB2a is relatively better. Secondly, the baseline performance outperforms the others which is normal because the matching method without pre-alignment nearly tried to use each minutia as the reference point. That is to say, the baseline incorporates the matching methods with pre-aligned by detected reference points. At last, when FMR $\approx 10^0$, the value of FNMR of our walking algorithm is very large which is caused by its numerous detection failures, and so it is with model-based method. As discussed before, the walking algorithm is unfairly forced to compare with other methods, so its weakest performance on ROC is understandable. Our EMS-based method and WEMS method have almost the same performance and both outperform mean-shift-based method and model-based method.

The EERs (Equal Error Rate, computed as the point where $\mathrm{FMR}(t) = \mathrm{FNMR}(t)$) of different methods are shown in Table 4. The reason of the baseline outperforming the others has been discussed in the last paragraph, so we do not compare it with the other methods again. The average EER value for the combined WEMS method is 4.30 and for mean-shift-based method is 7.70. That is to say, the proposed method has the best accuracy and achieves improvement of 3.40% in performance over the best of the other two prominent methods. Those matching methods with minutiae pre-aligned by reference points are usually much more efficient, despite the inaccuracy. Consequently, for fingerprint identification system with very large database that can tolerate EER $< 5\%$, the proposed WEMS method is the best choice to improve its efficiency.



(a) (b)

Fig. 6: ROCs of mean-shift-based method [1], model-based method [27] and our three algorithms on (a) FVC2000 DB1a and (b) FVC2000 DB2a

Table 4: EER (%) of the matching method without pre-alignment (the baseline) and with pre-aligned by reference points detected by different methods on FVC2000 DB2a and DB2a

Methods	DB1a	DB2a	Average
Baseline	2.14	1.77	1.96
Mean-shift-based [1]	9.89	5.50	7.70
Model-based [27]	11.39	6.86	9.13
Walking(ours)	17.23	13.45	15.34
EMS-based(ours)	5.54	**3.21**	4.38
WEMS(ours)	**5.04**	3.56	**4.30**

6 Conclusions and future works

This paper proposed a walking algorithm to navigate directly to singular point on extended directional field (EDF) without canning the fingerprint image. It is simple, efficient and easy to implement. Once the singular point is located by the walking algorithm, the EMS-based method, constructed by modifying the existing mean-shift-based method to a more accurate version, is utilized to localize the precise position of the reference point in the local area of the detected singular point. By combining the walking and EMS-based method, a fast and accurate method termed WEMS method is obtained. The combined WEMS method integrates the efficiency of walking algorithm and the accuracy of EMS-based method, achieving the best performance in comparison with two existing methods. The experimental results demonstrate that the proposed method is ready to be used in real time fingerprint identification systems.

Future works to improve the proposed method include: (i) redesigning the walking algorithm based on further analysis of the orientation field to make it a stand-alone method for singular point (including delta point) detection, (ii) exploring other strategy of choosing start points in the walking algorithm and (iii) defining a direction for the reference point using the topology of weighted cross points image.

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